

# Mono-Modal Waveguides Filled with a Pair of Parallel Epsilon-Negative (ENG) and Mu-Negative (MNG) Metamaterial Layers

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**Abstract** — Here we analyze guided wave propagation in a parallel-plate waveguide filled with a pair of parallel lossless slabs; one possessing negative real permittivity but positive real permeability, and the other with negative real permeability and positive real permittivity, in the range of frequency of interest. It is shown that such a waveguide can support only a single propagating mode, essentially independent of the total thickness of this structure. Furthermore, this waveguide can still possess a propagating mode even when its thickness is very small. Field distribution and dispersion relations in such a mono-modal waveguide are obtained and discussed with physical insights and intuitive description for the mathematical findings.

## I. INTRODUCTION

The electromagnetics of complex media has been the subject of interest for many researchers in the past several decades. In recent years, the topic of metamaterials, i.e., artificial materials synthesized by embedding specific inclusions in host media, has increasingly received a renewed attention due to the interest in man-made complex materials that may possess negative real permittivity and permeability in a certain range of frequency. In 1967, Veselago postulated theoretically a material in which both permittivity and permeability were assumed to have negative real values, and he analyzed plane wave propagation in such a medium, which he called “left-handed (LH)” medium [1]. According to his analysis, in such a “double-negative (DNG)” [2] medium the Poynting vector of a plane wave is antiparallel with its phase velocity. In recent years, Smith, Schultz and Shelby from UC San Diego [3], inspired by the work of Pendry ([4]), constructed such a LH composite medium in the microwave regime, by arranging arrays of small metallic wires and split ring resonators [3]. Various aspects of this class of metamaterials are now being studied by several groups worldwide, and many ideas and suggestions for potential applications of these media have been mentioned.

As one such idea, we theoretically suggested the possibility of having thin, subwavelength cavity resonators in which a layer of the DNG medium is paired with a layer of conventional material (i.e., a “double-positive (DPS)” medium) [5]-[6]. By exploiting the antiparallel nature of the phase velocity and Poynting vectors in a DNG slab, we found possibility of resonant modes and guided waves in very thin parallel-plate structures containing such a DNG-DPS bilayer structures [5]-[6].

The materials in which only one of the material parameters has negative value have also been of interest. These “single-negative (SNG)” media include the epsilon-negative (ENG) media, where the real part of the permittivity is negative but the real permeability is positive, and the mu-negative (MNG) media, in which real part of permeability is negative but real permittivity is positive. For instance, the idea of constructing an effective LH medium by having layers of SNG media has been explored in [7]. We have also analyzed the wave interaction with a pair of juxtaposed ENG and MNG slabs, showing interesting properties such as resonance, complete tunneling, zero reflection and transparency [8]. Using appropriate transmission-line model for the ENG-MNG paired layers, we have explained the unusual field behavior in these paired structures, and have shown that such a lossless pair may exhibit resonance phenomena, even though each slab alone does not manifest such an effect [8].

As a further contribution in this topic, here we present the results of our theoretical analysis on possibility of achieving mono-modal propagation in an arbitrarily thick parallel-plate waveguide containing an ENG-MNG parallel bilayered stack. We also show that such a guided-wave structure has no cut-off thickness.

As an aside, it is important to note that passive metamaterials with negative permittivity or negative permeability are inherently dispersive. So although  $\epsilon$  or

$\mu$  of such media can be negative at a certain band of frequency, these parameters do vary with frequency. Thus in general one should take into account the frequency dependence of such material parameters. However, this would be beyond the scope of the present work, where we want to emphasize the salient features of this type of waveguides without resorting to an unnecessary complexity. Therefore, for the remainder of this paper, we assume that we operate at a single frequency for which the material parameters are given and fixed.

### I. GEOMETRY OF THE ENG-MNG WAVEGUIDE

Let us consider a parallel-plate waveguide, made of two infinitely extent perfectly conducting plates separated by the distance  $d = d_1 + d_2$ , as shown in Fig. 1. This waveguide is filled with two parallel slabs of metamaterials; one being a lossless ENG material ( $\epsilon_1 < 0$ ,  $\mu_1 > 0$ ) with thickness  $d_1$  and the other being a lossless MNG material ( $\epsilon_2 > 0$ ,  $\mu_2 < 0$ ) with thickness  $d_2$ . The Cartesian coordinate system ( $x$ ,  $y$ ,  $z$ ) is shown in Fig. 1.

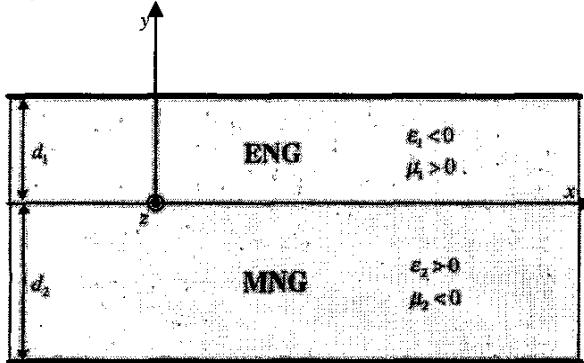


Fig. 1. Geometry of the parallel-plate waveguide filled with two parallel slabs, one being a lossless epsilon-negative (ENG) medium and the other being a lossless mu-negative (MNG) medium.

For the  $TE^x$  mode excitation, the following expressions for the fields can be written:

$$\mathbf{E}^{TE} = \hat{z} E_0 e^{-j\beta^{TE} x} \begin{cases} \sinh(k_{i2}^{TE} d_2) \sinh k_{i1}^{TE} (d_1 - y) & y > 0 \\ \sinh(k_{i1}^{TE} d_1) \sinh k_{i2}^{TE} (y + d_2) & y < 0 \end{cases} \quad (1)$$

$$\mathbf{H}^{TE} = \hat{x} \frac{jE_0 e^{-j\beta^{TE} x}}{\omega} \begin{cases} -\mu_1^{-1} k_{i1}^{TE} \sinh(k_{i2}^{TE} d_2) \cosh k_{i1}^{TE} (d_1 - y) \\ \mu_2^{-1} k_{i2}^{TE} \sinh(k_{i1}^{TE} d_1) \cosh k_{i2}^{TE} (y + d_2) \end{cases}$$

$$-\hat{y} \frac{\beta E_0 e^{-j\beta^{TE} x}}{\omega} \begin{cases} \mu_1^{-1} \sinh(k_{i2}^{TE} d_2) \sinh k_{i1}^{TE} (d_1 - y) \\ \mu_2^{-1} \sinh(k_{i1}^{TE} d_1) \sinh k_{i2}^{TE} (y + d_2) \end{cases} \quad (2)$$

where  $E_0$  is the mode amplitude, determined by the excitation, and  $k_i = \sqrt{|k_i|^2 + \beta^{TE 2}}$  with  $k_i^2 = \omega^2 \mu_i \epsilon_i < 0$  (since  $\mu_i \epsilon_i < 0$  in ENG and MNG slabs) for  $i = 1, 2$ . Satisfying the boundary conditions at the interface  $y = 0$ , we find the following dispersion relation for the TE case:

$$\frac{\mu_1}{k_{i1}^{TE}} \tanh(k_{i1}^{TE} d_1) = -\frac{\mu_2}{k_{i2}^{TE}} \tanh(k_{i2}^{TE} d_2). \quad (3)$$

The corresponding dispersion relation for the  $TM^x$  case can be found as:

$$\frac{\epsilon_1}{k_{i1}^{TM}} \coth(k_{i1}^{TM} d_1) = -\frac{\epsilon_2}{k_{i2}^{TM}} \coth(k_{i2}^{TM} d_2). \quad (4)$$

The above dispersion relations are of course general and valid for any two slabs with arbitrary complex values for the material parameters. For the case at hand where one slab is lossless ENG and the other is lossless MNG, however, our aim is to find the conditions under which we obtain real-valued solutions for the longitudinal wave number  $\beta$ . (In the following we refer to the TE excitation and the superscript TE is hereto after dropped.)

### II. DISPERSION CHARACTERISTICS

Rewriting Eq. (3) more explicitly for the ENG-MNG waveguide of Fig. 1, we obtain:

$$\frac{\mu_1}{\sqrt{|k_1|^2 + \beta^2}} \tanh\left(\sqrt{|k_1|^2 + \beta^2} d_1\right) = -\frac{\mu_2}{\sqrt{|k_2|^2 + \beta^2}} \tanh\left(\sqrt{|k_2|^2 + \beta^2} d_2\right) \quad (5)$$

Due to the monotonic behavior and limited variation of the hyperbolic tangent functions with real argument in Eq. (5), we expect to observe interesting dispersion characteristics. First, we notice that if for the TE case  $\mu_1$  and  $\mu_2$  have the same sign, Eq. (5) cannot have any real-valued solution for  $\beta$ . This is not surprising, since for the case of the ENG-ENG pair or the MNG-MNG pair, where  $\mu_1$  and  $\mu_2$  have the same sign, the wave number in the bulk paired materials is always imaginary. However, when we have a so-called “conjugate” pair, i.e., a pair of ENG and MNG slabs, in this waveguide, both sides of Eq. (5) have the same sign, and thus it is possible to have a

real-valued solution for  $\beta$  from Eq. (5). This is consistent with our previous findings on free-space plane wave interaction with a pair of ENG-MNG slabs [8], in which we have shown the interesting “resonance” behavior between the two slabs. In order to gain some physical insights into the possible real-valued solutions of Eq. (5), let us assume that the parameters of the ENG and MNG slabs, i.e.,  $\mu_1$ ,  $\epsilon_1$ ,  $\mu_2$ , and  $\epsilon_2$  are given at the frequency of interest, and that the thickness of the ENG slab  $d_1$  is known and fixed. We then wish to find  $d_2$  such that  $\beta$  attains a specific real value. This can be found by rearranging Eq. (5) as

$$d_2 \approx \frac{\tanh^{-1} \frac{|\mu_1| \sqrt{|\mathbf{k}_1|^2 + \beta^2}}{|\mu_2| \sqrt{|\mathbf{k}_1|^2 + \beta^2}} \tanh \left( \sqrt{|\mathbf{k}_1|^2 + \beta^2} d_1 \right)}{\sqrt{|\mathbf{k}_2|^2 + \beta^2}}. \quad (6)$$

Obviously, a physical solution for  $d_2$  exists only if the argument of the inverse hyperbolic tangent function in the above equation is less than unity. But when this condition is fulfilled, the solution for  $d_2$  is unique due to the monotonic (non-periodic) behavior of the hyperbolic tangent function. Specifically, for a given set of parameters for the ENG and MNG slabs and a fixed  $d_1$ , there may only be one value for  $d_2$  if a specific real value for  $\beta$  is desired. Furthermore, the field components vary as hyperbolic sinusoidal functions as shown in Eqs. (1) and (2), and they are mostly concentrated around the ENG-MNG interface. These features are in sharp contrast with those of propagating modes in a conventional waveguide filled with a pair of DPS slabs or a waveguide filled with the DPS-DNG paired slabs [5],[6], where one can have many solutions for  $d_2$  due to the multi-valuedness/periodic behavior of the interior fields on the transverse plane of the waveguide.

Figure 2 presents the TE dispersion diagram of such an ENG-MNG waveguide for two different sets of material parameters. One striking feature of these diagrams, as already underlined, is the single-valuedness of  $\beta$  for a given set of  $d_1$  and  $d_2$ , unlike the case of ordinary waveguides where one may find multiple real values for wave number for a given dimension of a waveguide and the corresponding figure would be multiple-branched. In order to discuss these figures, we first analyze some special limits. At one extreme, let us assume  $d_1$  and  $d_2$  to be very large. Eq. (5) will then be simplified as

$$\frac{|\mu_1|}{\sqrt{|\mathbf{k}_1|^2 + \beta^2}} \approx \frac{|\mu_2|}{\sqrt{|\mathbf{k}_2|^2 + \beta^2}}, \quad (7)$$

which is independent of the thicknesses of the slabs and has the following solutions for  $\beta$ :

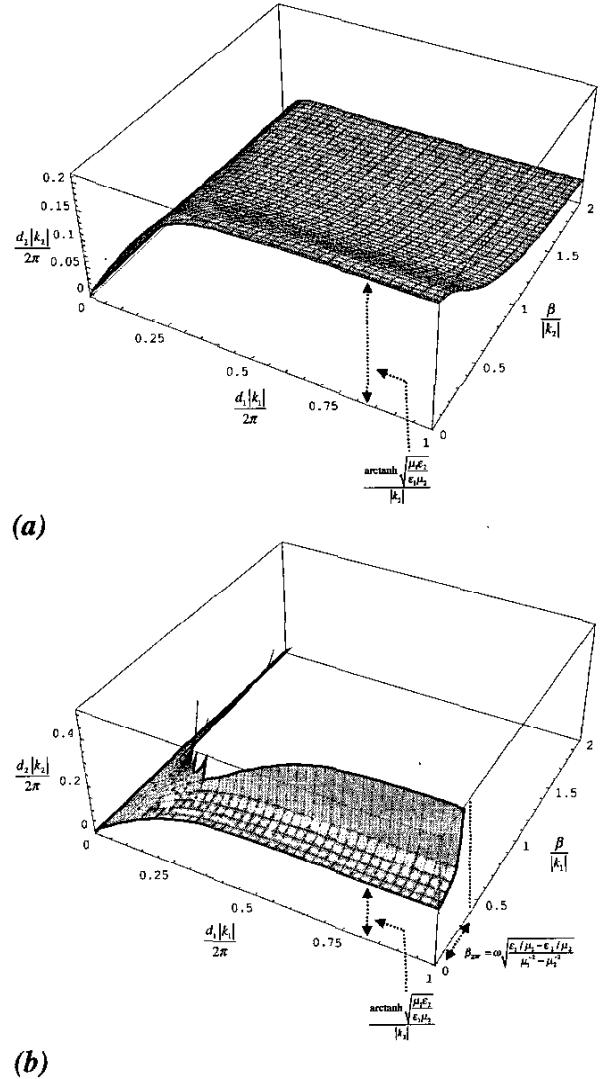


Fig. 2. Dispersion diagram presenting the relationship among  $d_1$ ,  $d_2$ , and real-valued  $\beta$ , as described in Eq. (5), for two sets of parameters at a fixed frequency of interest: (a) when  $\epsilon_1 = -5\epsilon_0$ ,  $\mu_1 = 2\mu_0$ ,  $\epsilon_2 = 2\epsilon_0$ ,  $\mu_2 = -\mu_0$ , and (b) when  $\epsilon_1 = -2\epsilon_0$ ,  $\mu_1 = \mu_0$ ,  $\epsilon_2 = 3\epsilon_0$ ,  $\mu_2 = -2\mu_0$ .

$$\beta = \omega \sqrt{\frac{\epsilon_1/\mu_1 - \epsilon_2/\mu_2}{\mu_1^{-2} - \mu_2^{-2}}}. \quad (8)$$

With proper choices of material parameters,  $\beta$  can be a real-valued quantity, and then it represents the wave number of the Zenneck wave that can exist along the interface of the two semi-infinite lossless ENG and MNG media. This can be seen in Fig. 2b, in which the material parameters are chosen such that the Zenneck wave can

exist. We note that for the limit of very large  $d_1$  and  $d_2$ , the wavenumber  $\beta$  is shown to approach the value given in Eq. (8). It is interesting to note that this expression is identical to the corresponding wave number we addressed in [9] for the Zenneck wave along a DPS-DNG interface.

Another limit is for the case of thin layers. If the thicknesses  $d_1$  and  $d_2$  are taken to be very small, Eq. (5) will be expressed approximately as

$$d_2/d_1 = |\mu_1|/|\mu_2|, \quad (9)$$

an expression that is effectively independent of  $\beta$ . In such a limit, the wave number of the guided mode can essentially be any real value, as can be seen in Figs. 2a and 2b around the region where  $d_1 \rightarrow 0$  and  $d_2 \rightarrow 0$ . In such a waveguide, no matter how thin these layers are (as long as they satisfy Eq. (9)), one (and only one) propagating mode can exist. In other words, this waveguide does not have a “cut-off” thickness below which no mode can propagate, analogously to a DPS-DNG thin cavity [5].

Another interesting feature to note in Fig. 2 is that for  $\beta = 0$ , which is the case of a 1-dimensional cavity filled with the ENG-MNG pair, if  $d_1$  is chosen to be large,  $d_2$  will approach a finite limit given as

$$d_2 = \frac{1}{|k_2|} \tanh^{-1} \sqrt{\frac{\mu_1 \epsilon_2}{\mu_2 \epsilon_1}}. \quad (10)$$

Depending on the choice of the material parameters, for a fixed  $d_1$ , as  $\beta$  increases from zero the thickness  $d_2$  either becomes greater than the value shown in Eq. (10) or less than it, as can be seen in Figs. 2a and 2b, respectively. Moreover, it is important to underline that the mono-modal characteristics of such a waveguide is strikingly independent of its total thickness. For instance, from Fig. 2a, we can see that for a given  $\beta$  and a specific allowable  $d_2$ , thickness  $d_1$  can be chosen arbitrarily large, resulting in a thick waveguide. But still one single mode is propagating in such a thick waveguide. This feature, not present in a conventional waveguide, for which increasing the thickness leads to the possibility of having multiple propagating modes, can be potentially employed for some applications. Finally, one notes that in the dispersion diagrams in Fig. 2b there is a curved line beyond which the diagram “stops”. This is due to the fact that beyond this boundary, the real solution for Eq. (6) does not exist.

## V. CONCLUSION

Our analysis has shown that a parallel-plate waveguide filled with a pair of parallel layers of lossless epsilon-negative and mu-negative materials may possess only a

single propagating mode with essentially no restriction on the waveguide thickness. So a thick waveguide of this kind is always mono-modal, contrary to a conventional waveguide. In the limit of small thickness, this waveguide may always have a propagating mode, thus has no cut-off thickness.

Such a waveguide may find interesting applications in design of single-mode fibers with less restriction and more flexibility on the fiber thickness.

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